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## **Multi-Mission Selective Maintenance Decisions**

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## Executive Summary

All military organizations depend on the reliable performance of repairable systems for the successful completion of missions. Due to limitations in maintenance resources, a maintenance manager must decide how to allocate available resources. This allocation falls within the domain of selective maintenance. Selective maintenance is defined as the process of identifying the subset of maintenance activities to perform from a set of desired maintenance actions. Previously, researchers have developed a class of mathematical models that can be used to identify selective maintenance decisions for the following scenario – A system has just completed a mission and will begin its next mission soon. Maintenance cannot be performed during missions, therefore, the decision-maker must decide which components to maintain prior to the next mission. The selective maintenance models considered to date treat decision-making relative to a single, future mission. If a system is required to perform a sequence of missions, then the selective maintenance decisions directly affect system reliability for the next mission and indirectly affect the system reliability for later missions.

The primary objective of this project is to develop a modeling-based methodology for managing selective maintenance decisions when the planning horizon is more than one future mission. First, the research literature for selective maintenance is presented. The selective maintenance literature is limited in that current models only consider a single, future mission. Next, background research is presented in which the selective maintenance model treating decision-making relative to a single, future mission is defined. This model serves as the foundation for our multi-mission analysis.

In this work, we define a scenario in which a system must perform a sequence of missions. The reliability characteristics of the system are defined, and we extend the single-



mission scenario parameters such that decision variables for one mission are the input parameters for the next mission. An objective function that maximizes the expected number of successful missions remaining in the planning horizon is defined. Finally, we formulate a stochastic dynamic programming model to solve the multi-mission scenario and present a numerical example. This example shows that the selective maintenance decisions relative to a multi-mission scenario may differ from the decisions for a single-mission scenario.

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## 1. Introduction

All military organizations depend on the reliable performance of repairable systems for the successful completion of missions. The use of mathematical modeling for the purpose of modeling repairable systems and designing optimal maintenance policies for these systems has received an extensive amount of attention in the literature. Unfortunately, the vast majority of this work ignores potential limitations on the resources required to perform maintenance actions. This shortcoming has motivated the development of models for selective maintenance, the process of identifying the subset of actions to perform from a set of desirable maintenance actions. Previously, researchers have developed a class of mathematical models that can be used to identify selective maintenance decisions for the following scenario – A system has just completed a mission and will begin its next mission soon. Maintenance cannot be performed during missions, therefore, the decision-maker must decide which components to maintain prior to the next mission. The selective maintenance models considered to date treat decision-making relative to a single, future mission. If a system is required to perform a sequence of missions, then the selective maintenance decisions directly affect system reliability for the next mission and indirectly affect the system reliability for later missions. The primary objective of this project is to develop a modeling-based methodology for managing selective maintenance decisions when the planning horizon is more than one future mission.

Achieving the objective of this project requires the completion of several key activities. First, we modify existing selective maintenance models into a multi-mission formulation. To complete this activity, we extend the scenario parameters such that decision variables for one mission are the input parameters for the next mission. Second, we define an objective function that maximizes the expected number of successful missions over the planning horizon. Third,

we formulate a stochastic dynamic programming model to solve the multi-mission scenario. We then use an enumerative approach to determine the optimal selective maintenance decisions. Finally, we demonstrate the multi-mission scenario using a numerical example. This example shows that the selective maintenance decisions relative to a multi-mission scenario may differ from the decisions for a single-mission scenario.

## 2. Research Literature Review

This project builds upon the body of knowledge in selective maintenance. Selective maintenance falls within the domain of maintenance modeling and optimization. The use of mathematical modeling for the purpose of modeling repairable systems and designing optimal maintenance policies for these systems has received an extensive amount of attention in the literature [5, 6, 7, 8, 11, 12, 13].

The original study in selective maintenance was performed by Rice *et al.* [9]. They define a system that must complete a series of missions where maintenance is performed only during finite breaks between missions. Due to the limited maintenance time, it may not be possible to repair all failed components before the next mission. A nonlinear, discrete selective maintenance optimization model is developed which is designed to maximize system reliability for the next mission. The numbers of components to repair are the decision variables, and the limitation on maintenance time serves as the primary functional constraint. Due to the complexity of the model, total enumeration is the recommended solution procedure. Given that total enumeration is ineffective for large scenarios, a heuristic selective maintenance procedure is developed.

Cassady *et al.* [1, 2] extend the work of Rice *et al.* [9] in several ways. First, more complex systems are analyzed. Specifically, systems are comprised of independent subsystems connected in series with the individual components in each subsystem connected in any fashion. Next, the selective maintenance model is extended to consider the case where both time and cost are constrained. This leads to the development of three different selective maintenance models. These models include maximizing system reliability subject to both time and cost constraints;

minimizing system repair costs subject to a time constraint and a minimum required reliability level; and minimizing total repair time subject to both cost and reliability constraints.

Cassady *et al.* [3] extend the work of Rice *et al.* [9] in two other ways. First, system components are assumed to have Weibull life distributions. This assumption permits systems to experience an increasing failure rate (IFR) and requires monitoring of the age of components. Second, the selective maintenance model is formulated to include three maintenance actions: minimal repair of failed components, replacement of failed components, and preventive maintenance.

Chen *et al.* [4] extend the work of Rice *et al.* [9] and Cassady *et al.* [1] by considering systems in which each component and the system may be in  $K + 1$  possible states,  $0, 1, \dots, K$ . They use an optimization model to minimize the total cost of maintenance activities subject to a minimum required system reliability.

Schneider and Cassady [10] formulate an optimization model to extend the work of Rice *et al.* [9] by defining a selective maintenance model for a set of systems that must perform a set of missions with system maintenance performed only between sets of missions. Three models are formulated. The first model maximizes the probability that all systems within the set successfully complete the next mission, where as the second model minimizes the variable cost associated with maintenance. A special case of the second model allows the user to maximize the expected value of the number of successful missions in the next set. The third model permits cancellation of a mission based on costs associated with the risk of failure.

### 3. Background Research

In this section, the selective maintenance model treating decision-making relative to a single, future mission is defined. This model serves as the foundation for our multi-mission analysis.

#### 3.1 Hypothetical System

Consider a system comprised of  $m$  independent subsystems connected in series, and suppose subsystem  $i$  contains  $n_i$  independent and identical copies of a constant failure rate (CRF) component connected in parallel. A graphical representation of a hypothetical system is given in Figure 3.1. Note that each component in the system is labeled  $(i,j)$  where  $i$  denotes the subsystem number and  $j$  denotes the component number. At any point in time, a component is either functioning (1) or failed (0). Note that subsystems and the system also have binary status.

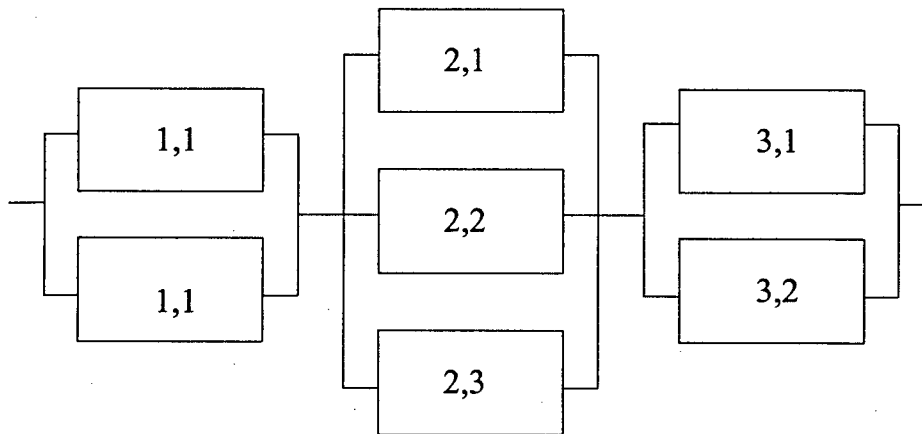


Figure 3.1 Hypothetical system

#### 3.2 System Performance

The system is required to perform a sequence of identical missions with finite breaks between each mission. Failures only occur during missions, and maintenance is performed only during breaks between missions. Let  $r_i$  denote the mission reliability of a component in subsystem  $i$  that is functioning at the start of a mission. The reliability of component  $ij$  is defined

to be the probability that component  $ij$  is functioning at the end of the next mission and is given by

$$R_{ij} = \begin{cases} r_i & \text{if component } ij \text{ is functioning at the start of the next mission} \\ 0 & \text{otherwise} \end{cases} \quad (3.1)$$

Likewise, the reliability of subsystem  $i$  is the probability that subsystem  $i$  is functioning at the end of the next mission. Since each subsystem is a parallel arrangement of its components, subsystem reliability is given by

$$R_i = \prod_{j=1}^{n_i} R_{ij} = 1 - \prod_{j=1}^{n_i} (1 - R_{ij}) \quad (3.2)$$

Let  $b_i$  denote the number of functioning components in subsystem  $i$  at the beginning of the next mission. Since each subsystem contains identical copies of CFR components, equation (3.2) can be simplified to

$$R_i = 1 - (1 - r_i)^{b_i} \quad (3.3)$$

Finally, the reliability of the system is defined to be the probability that all subsystems are functioning at the end of the next mission. Since the system is a series arrangement of its subsystems, system reliability is given by

$$R = \prod_{i=1}^m R_i \quad (3.4)$$

which can be re-written as

$$R = \prod_{i=1}^m 1 - (1 - r_i)^{b_i} \quad (3.5)$$

### 3.3 The Selective Maintenance Model for a Single Future Mission

Recall that we consider the scenario where a mission has just ended and the system has returned to its base of operation and maintenance. Ideally, all failed components are repaired



prior to the beginning of the next mission. However, limitations on maintenance resources may prevent the repair of all failed components.

Let  $a_i$  denote the number of failed components in subsystem  $i$  at the end of the next mission. Each repair of a failed component consumes a fixed amount of  $s$  limited maintenance resources. Let  $\alpha_{il}$  denote the amount of resource  $l$  required to repair a component in subsystem  $i$ . Let  $\beta_l$  denote the amount of resource  $l$  available during a single break. If

$$\sum_{i=1}^m \alpha_{il} a_i > \beta_l \quad (3.6)$$

for some  $l \in \{1, 2, \dots, s\}$ , then sufficient resources do not exist to repair all failed components prior to the next mission. In such a case, a method is needed to decide which failed components to repair during the maintenance break. This is the decision-making process considered for the selective maintenance scenario relative to a single future mission.

Let  $d_i$  denote the number of failed components in subsystem  $i$  to be repaired prior to the beginning of the next set of missions. Rice *et al.* [9] formulate a nonlinear, discrete selective maintenance optimization model to maximize system reliability for the next mission. The numbers of components to repair during the maintenance break serve as the decision variables. The first constraint on the decision variables is that they be integer-valued. Second, the number of repairs is limited to the number of failed components, i.e.

$$0 \leq d_i \leq a_i, \text{ integer} \quad i = 1, 2, \dots, m \quad (3.7)$$

Finally, the number of repairs is limited by the available maintenance resources, i.e.

$$\sum_{i=1}^m \alpha_{il} d_i \leq \beta_l \quad l = 1, 2, \dots, s \quad (3.8)$$

The objective in choosing values for the decision variables is to maximize the system reliability for the upcoming mission. The full formulation of the selective maintenance model relative to a single, future mission is given by

$$\begin{aligned}
 \mathbf{P} \quad & \text{Maximize} \quad R = \prod_{i=1}^m 1 - (1 - r_i)^{b_i} \\
 & \text{s.t.} \quad \sum_{i=1}^m \alpha_{il} d_i \leq \beta_l \quad l = 1, 2, \dots, s \\
 & \quad \quad 0 \leq d_i \leq a_i, \text{ integer} \quad i = 1, 2, \dots, m
 \end{aligned}$$

#### 4. Multi-Mission Selective Maintenance

If a system is required to perform a sequence of missions, the selective maintenance decisions made during a maintenance break directly affect system reliability for the next mission and indirectly affect the system reliability for later missions. In this section, the selective maintenance model from Section 3 is extended to treat decision-making relative to more than one future mission.

##### 4.1 Hypothetical System

Consider a system comprised of  $m$  independent subsystems connected in series, and suppose subsystem  $i$  contains  $n_i$  independent and identical copies of a constant failure rate (CFR) component connected in parallel. Let  $\underline{n}$  be a vector denoting the number of components in each subsystem, i.e.  $\underline{n} = [n_1, n_2, \dots, n_m]$ . A graphical representation of an example system is given in Figure 4.1. Note that each component in the system is labeled  $(i,j)$  where  $i$  denotes the subsystem number and  $j$  denotes the component number. For this system,  $\underline{n} = [3, 4, 2]$ . At any point in time, a component is either functioning (1) or failed (0). Note that subsystems and the system also have binary status.

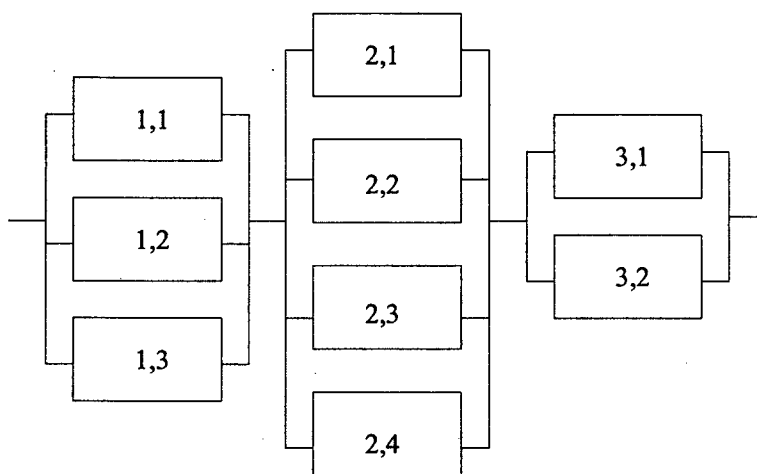


Figure 4.1 Hypothetical system

## 4.2 System Performance

The system is required to perform a sequence of identical missions with finite breaks between each mission. Failures only occur during missions, and maintenance is performed only during breaks between missions. Let  $r_i$  denote the mission reliability of a component in subsystem  $i$  that is functioning at the start of a mission. Let  $b_i$  denote the number of functioning components in subsystem  $i$  at the beginning of the next mission, and let  $\underline{b} = [b_1, b_2, \dots, b_m]$ . The reliability of subsystem  $i$  is the probability that subsystem  $i$  is functioning at the end of the next mission. Since each subsystem is a parallel arrangement of identical CFR components, subsystem reliability is given by

$$R_i = 1 - (1 - r_i)^{b_i} \quad (4.1)$$

Finally, the reliability of the system is defined to be the probability that all subsystems are functioning at the end of the next mission. Since the system is a series arrangement of its subsystems, system reliability is given by

$$R = \prod_{i=1}^m 1 - (1 - r_i)^{b_i} \quad (4.2)$$

Let  $A'_i$  denote the number of failed components in subsystem  $i$  at the end of the next mission. Note that  $A'_i$  is a random variable given by

$$A'_i = n_i - b_i + Z_i \quad (4.3)$$

where  $Z_i$  is the number of component failures in subsystem  $i$  during the next mission. Note that  $Z_i$  is a binomial random variable with  $b_i$  individual and identical Bernoulli trials having probability of success  $1 - r_i$ , i.e.

$$Z_i \sim \text{bin}(b_i, 1 - r_i) \quad (4.4)$$

This implies that

$$A'_i \in \{n_i - b_i, n_i - b_i + 1, \dots, n_i\} \quad (4.5)$$

Let  $\underline{A}' = [A'_1, A'_2, \dots, A'_m]$ , and let  $a'_i$  denote a specific realization of  $A'_i$ . Also let  $\underline{a}' = [a'_1, a'_2, \dots, a'_m]$ . Then,

$$\Pr(A'_i = a'_i) = \Pr(n_i - b_i + Z_i = a'_i) = \Pr(Z_i = a'_i - n_i + b_i) \quad (4.6)$$

$$\Pr(A'_i = a'_i) = b(a'_i - n_i + b_i, b_i, 1 - r_i) = \binom{b_i}{a'_i - n_i + b_i} (1 - r_i)^{a'_i - n_i + b_i} (r_i)^{n_i - a'_i} \quad (4.7)$$

Therefore,

$$\Pr(\underline{A}' = \underline{a}') = \prod_{i=1}^m b(a'_i - n_i + b_i, b_i, 1 - r_i) = f(\underline{a}' | \underline{b}) \quad (4.8)$$

### 4.3 The Multi-Mission Selective Maintenance Model

Consider the scenario where a mission has just ended and the system has returned to its base of operation and maintenance. Let  $t$  denote the number of missions remaining in the planning horizon. Ideally, all failed components are repaired prior to the beginning of the next mission. However, limitations on maintenance resources may prevent the repair of all failed components.

The number of failed components in subsystem  $i$  is denoted by  $a_i$ . Let  $\underline{a} = [a_1, a_2, \dots, a_m]$ . Each repair of a failed component consumes a fixed amount of  $s$  limited maintenance resources. Let  $\alpha_{il}$  denote the amount of resource  $l$  required to repair a component in subsystem  $i$ , and let  $\beta_l$  denote the amount of resource  $l$  available during a single break. Let

$$\underline{\alpha} = \begin{bmatrix} \alpha_{11}, \alpha_{12}, \dots, \alpha_{1s} \\ \alpha_{21}, \alpha_{22}, \dots, \alpha_{2s} \\ \vdots \\ \alpha_{m1}, \alpha_{m2}, \dots, \alpha_{ms} \end{bmatrix} \quad (4.9)$$

and

$$\underline{\beta} = [\beta_1, \beta_2, \dots, \beta_m] \quad (4.10)$$

If

$$\underline{a} \leq \underline{\beta} \quad (4.11)$$

then sufficient resources exist to repair all failed components prior to the next mission. Otherwise, a method is needed to decide which failed components to repair during the maintenance break.

Let  $d_i$  denote the number of failed components in subsystem  $i$  to be repaired prior to the beginning of the next set of missions, and let  $\underline{d} = [d_1, d_2, \dots, d_m]$ . To address the selective maintenance issue, we formulate a stochastic dynamic programming model where the  $\underline{d}$  values serve as the decision variables. The first constraint on these decision variables is that they be integer-valued. Second, the number of repairs is limited to the number of failed components. Third, the number of repairs is limited by the available maintenance resources. Let  $D(\underline{a})$  denote the set of feasible selective maintenance decisions. Then  $\underline{d} \in D(\underline{a})$  iff

$$0 \leq d_i \leq a_i, \text{ integer } i = 1, 2, \dots, m$$

and

$$\underline{d} \leq \underline{\beta} \quad (4.12)$$

Recall that  $b_i$  denotes the number of functioning components in subsystem  $i$  at the beginning of the next mission, and note that

$$b_i = n_i - a_i + d_i \quad (4.13)$$

which implies that

$$\underline{b} = \underline{n} - \underline{a} + \underline{d} \quad (4.14)$$

Recall that  $A'_i$  is the number of failed components in subsystem  $i$  at the end of the next mission, and  $a'_i$  is a specific realization of  $A'_i$ . Substituting equation (4.13) into equation (4.3) yields

$$A'_i = a_i - d_i + Z_i \quad (4.15)$$

where

$$Z_i \sim \text{bin}(n_i - a_i + d_i, 1 - r_i) \quad (4.16)$$

which implies that

$$A'_i \in \{a_i - d_i, a_i - d_i + 1, \dots, n_i\} \quad (4.17)$$

This also means that the probability of a specific realization of  $A'_i$  can be re-written as

$$\Pr(A'_i = a'_i) = \Pr(a_i - d_i + Z_i = a'_i) = \Pr(Z_i = a'_i - a_i + d_i) \quad (4.18)$$

$$\Pr(A'_i = a'_i) = b(a'_i - a_i + d_i, n_i - a_i + d_i, 1 - r_i) = \binom{n_i - a_i + d_i}{a'_i - a_i + d_i} (1 - r_i)^{a'_i - a_i + d_i} (r_i)^{n_i - a'_i} \quad (4.19)$$

Therefore,

$$\Pr(\underline{A}' = \underline{a}') = \prod_{i=1}^m b(a'_i - a_i + d_i, n_i - a_i + d_i, 1 - r_i) = f(\underline{a}' | \underline{a}, \underline{d}) \quad (4.20)$$

The reliability of the system for the upcoming mission is a function of both  $\underline{a}$  and  $\underline{d}$ .

Therefore, system reliability for the next mission can be expressed as

$$R(\underline{a}, \underline{d}) = \prod_{i=1}^m 1 - (1 - r_i)^{n_i - a_i + d_i} \quad (4.21)$$

Let  $W(t, \underline{a})$  denote the maximum expected value of the number of successful missions with  $t$  missions remaining in the planning horizon and a system status of  $\underline{a}$ . Note that the number of successes in the next mission (which will be either 0 or 1) is a Bernoulli random variable having probability of success  $R(\underline{a}, \underline{d})$ . Since the model is formulated over a finite planning horizon,

when there are no future outstanding missions, the expected value of the number of successful missions remaining is equal to zero, i.e.

$$W(0, \underline{a}) = 0 \quad (4.22)$$

regardless of the value of  $\underline{a}$ . When a single mission remains,  $W(1, \underline{a})$  is equivalent to the maximum value of system reliability for the next mission, i.e.

$$W(1, \underline{a}) = \max_{\underline{d} \in D(\underline{a})} \{R(\underline{a}, \underline{d})\} \quad (4.23)$$

This corresponds to the original single-system, single-mission selective maintenance scenario described in Section 3. The optimal selective maintenance decisions for a given  $\underline{a}$  are denoted by  $\underline{d}^*(1, \underline{a})$ . This optimization problem must be solved for all  $\underline{a}$  and the corresponding  $W(1, \underline{a})$  values must be tabulated. Let

$$\Phi = \left\{ \underline{a} \mid \exists l \in \{1, 2, \dots, s\} \ni \sum_{i=1}^m \alpha_{il} a_i > \beta_l \right\} \quad (4.24)$$

i.e.  $\Phi$  is the set of all  $\underline{a}$  that require selective maintenance. Let  $R_{\max}$  denote the system reliability if the system is fully functioning after maintenance, i.e.

$$R_{\max} = \prod_{i=1}^m 1 - (1 - r_i)^{n_i} \quad (4.25)$$

Note that if  $\underline{a} \notin \Phi$ , then  $\underline{d}^*(1, \underline{a}) = \underline{a}$  and  $W(1, \underline{a}) = R_{\max}$ .

As a numerical example, consider the system presented in Figure 4.1 having  $m = 3$ . Three ( $s = 3$ ) maintenance resources limit repair activities during the break between mission sets. Note that  $\beta_1 = 12$ ,  $\beta_2 = 10$ , and  $\beta_3 = 12$ . The remaining parameters for the systems are given in Table 4.1. Also note that  $R_{\max} = 0.995998$ .



Table 4.1 System parameters

Subsystem	$n_j$	$r_j$	$\alpha_{i1}$	$\alpha_{i2}$	$\alpha_{i3}$
1	3	0.90	3	1	2
2	4	0.85	5	6	5
3	2	0.95	2	2	4

Suppose that the system returned to its base of operation and maintenance with two failed components in subsystems 1 and 2 and one failed component in subsystem 3, i.e.  $\underline{a} = [2, 2, 1]$ . The optimal selective maintenance actions for the original single-mission scenario are to repair one component in each subsystem, i.e.  $\underline{d}^*(1, \underline{a}) = [1, 1, 1]$ , resulting in a system reliability and expected value of the number of successful missions remaining of 0.98419, i.e.

$$W(1, \underline{a}) = R(\underline{a}, \underline{d}^*(1, \underline{a})) = 0.98419 \quad (4.26)$$

Table 4.2 contains the optimal selective maintenance decisions and the corresponding system reliabilities for all  $\underline{a} \in \Phi$  when  $t = 1$ . Note that for this example, there are 20 instances of  $\underline{a}$  for which  $\underline{d}^*(1, \underline{a}) = \underline{a}$ .

Table 4.2 Optimal selective maintenance decisions and  $W(1, \underline{a})$  values for all  $\underline{a} \in \Phi$

$\underline{a}$	$\underline{d}^*(1, \underline{a})$	$W(1, \underline{a})$	$\underline{a}$	$\underline{d}^*(1, \underline{a})$	$W(1, \underline{a})$
[0, 1, 2]	[0, 0, 2]	0.99314	[2, 2, 0]	[2, 1, 0]	0.99314
[0, 2, 0]	[0, 1, 0]	0.99314	[2, 2, 1]	[1, 1, 1]	0.98419
[0, 2, 1]	[0, 1, 1]	0.99314	[2, 2, 2]	[2, 0, 2]	0.97408
[0, 2, 2]	[0, 0, 2]	0.97408	[2, 3, 0]	[2, 1, 0]	0.97408
[0, 3, 0]	[0, 1, 0]	0.97408	[2, 3, 1]	[1, 1, 1]	0.96531
[0, 3, 1]	[0, 1, 1]	0.97408	[2, 3, 2]	[1, 1, 1]	0.91934
[0, 3, 2]	[0, 1, 1]	0.92770	[2, 4, 0]	[2, 1, 0]	0.84703
[0, 4, 0]	[0, 1, 0]	0.84703	[2, 4, 1]	[1, 1, 1]	0.83940
[0, 4, 1]	[0, 1, 1]	0.84703	[2, 4, 2]	[1, 1, 1]	0.79943
[0, 4, 2]	[0, 1, 1]	0.80669	[3, 0, 2]	[2, 0, 2]	0.98703
[1, 1, 2]	[1, 0, 2]	0.99314	[3, 1, 0]	[3, 0, 0]	0.99314
[1, 2, 0]	[1, 1, 0]	0.99314	[3, 1, 1]	[3, 0, 1]	0.99314
[1, 2, 1]	[1, 1, 1]	0.99314	[3, 1, 2]	[2, 0, 2]	0.98419
[1, 2, 2]	[1, 0, 2]	0.97408	[3, 2, 0]	[2, 1, 0]	0.98419
[1, 3, 0]	[1, 1, 0]	0.97408	[3, 2, 1]	[3, 0, 1]	0.97408
[1, 3, 1]	[1, 1, 1]	0.97408	[3, 2, 2]	[2, 0, 2]	0.96531
[1, 3, 2]	[1, 1, 1]	0.92770	[3, 3, 0]	[2, 1, 0]	0.96531
[1, 4, 0]	[1, 1, 0]	0.84703	[3, 3, 1]	[2, 1, 0]	0.91934
[1, 4, 1]	[1, 1, 1]	0.84703	[3, 3, 2]	[2, 0, 2]	0.83940
[1, 4, 2]	[1, 1, 1]	0.80669	[3, 4, 0]	[2, 1, 0]	0.83940

When two missions remain, the maximum expected value of the number of successful missions remaining is given by

$$W(2, \underline{a}) = \max_{\underline{d} \in D(\underline{a})} \left\{ R(\underline{a}, \underline{d}) + \sum_{\underline{a}'} W(1, \underline{a}') f(\underline{a}' | \underline{a}, \underline{d}) \right\} \quad (4.27)$$

The optimal selective maintenance decisions for a given  $\underline{a}$  in this scenario are denoted by  $\underline{d}^*(2, \underline{a})$ . This optimization problem must be solved for all  $\underline{a}$  and the corresponding  $W(2, \underline{a})$  values must be tabulated. If  $\underline{a} \notin \Phi$ , then  $\underline{d}^*(2, \underline{a}) = \underline{a}$  and

$$W(2, \underline{a}) = R_{\max} + \sum_{\underline{a}'} W(1, \underline{a}') f(\underline{a}' | \underline{a}, \underline{d}) \quad (4.28)$$

Again consider the scenario where the system has returned to its base of operation and maintenance with two failed components in subsystems 1 and 2 and one failed component in subsystem 3, i.e.  $\underline{a} = [2, 2, 1]$ . Sufficient resources do not exist to repair all failed components prior to the next mission. The feasible selective maintenance actions,  $\underline{d} \in D(\underline{a})$ , and the corresponding system reliabilities,  $R(\underline{a}, \underline{d})$  are shown in Table 4.3

Table 4.3 Feasible selective maintenance decisions and corresponding system reliabilities

$\underline{d}$	$R(\underline{a}, \underline{d})$
[0, 0, 0]	0.83576
[0, 0, 1]	0.87755
[0, 1, 0]	0.85211
[0, 1, 1]	0.89472
[1, 0, 0]	0.91934
[1, 0, 1]	0.96531
[1, 1, 0]	0.93733
[1, 1, 1]	0.98419
[2, 0, 0]	0.92770
[2, 0, 1]	0.97408
[2, 1, 0]	0.94585

If no failed components are repaired during the maintenance break, i.e.  $\underline{d} = [0, 0, 0]$ , the system will return from the next mission in one of twelve states. For example, the system may return from the next mission with no additional failed components, i.e.  $\underline{a}' = [2, 2, 1]$ . The probability that the system returns in this particular state is given by

$$\Pr(\underline{A}' = [2, 2, 1]) = f([2, 2, 1] | [2, 2, 1], [0, 0, 0]) \quad (4.29)$$

$$\Pr(\underline{A}' = [2, 2, 1]) = b(0, 1, 0.10) \times b(0, 2, 0.15) \times b(0, 1, 0.05) = 0.61774 \quad (4.30)$$

The twelve possible states and their corresponding probabilities of occurrence are shown in Table 4.4

Table 4.4 Possible states and corresponding probability of occurrence

$\underline{a}'$	$f(\underline{a}'   [2, 2, 1], [0, 0, 0])$
[2, 2, 1]	0.61774
[2, 2, 2]	0.03251
[2, 3, 1]	0.21803
[2, 3, 2]	0.01148
[2, 4, 1]	0.01924
[2, 4, 2]	0.00101
[3, 2, 1]	0.06864
[3, 2, 2]	0.00361
[3, 3, 1]	0.02423
[3, 3, 2]	0.00128
[3, 4, 1]	0.00214
[3, 4, 2]	0.00011

Table 4.5 contains the optimal selective maintenance decisions and the corresponding maximum expected value of the number of successful missions out of the two missions remaining for all  $\underline{a} \in \Phi$ . Note that for the scenario in which  $\underline{a} = [3, 3, 2]$ , the optimal selective maintenance actions for the two-mission scenario differ from the single mission scenario.

In general, when  $t$  missions remain in the planning horizon, the maximum expected value of the number of successful missions remaining is given by

$$W(t, \underline{a}) = \max_{\underline{d} \in D(\underline{a})} \left\{ R(\underline{a}, \underline{d}) + \sum_{\underline{a}'} W(t-1, \underline{a}') f(\underline{a}' | \underline{a}, \underline{d}) \right\} \quad (4.31)$$

where  $\underline{d}^*(t, \underline{a})$  denotes the optimal selective maintenance decisions.

Table 4.5 Optimal selective maintenance decisions and  $W(2, \underline{a})$  values for all  $\underline{a} \in \Phi$

$\underline{a}$	$\underline{d}^*(2, \underline{a})$	$W(2, \underline{a})$	$\underline{a}$	$\underline{d}^*(2, \underline{a})$	$W(2, \underline{a})$
[0, 1, 2]	[0, 0, 2]	1.98640	[2, 2, 0]	[2, 1, 0]	1.98640
[0, 2, 0]	[0, 1, 0]	1.98640	[2, 2, 1]	[1, 1, 1]	1.97733
[0, 2, 1]	[0, 1, 1]	1.98640	[2, 2, 2]	[2, 0, 2]	1.95897
[0, 2, 2]	[0, 0, 2]	1.95897	[2, 3, 0]	[2, 1, 0]	1.95897
[0, 3, 0]	[0, 1, 0]	1.95897	[2, 3, 1]	[1, 1, 1]	1.94997
[0, 3, 1]	[0, 1, 1]	1.95897	[2, 3, 2]	[1, 1, 1]	1.90118
[0, 3, 2]	[0, 1, 1]	1.91110	[2, 4, 0]	[2, 1, 0]	1.80190
[0, 4, 0]	[0, 1, 0]	1.80190	[2, 4, 1]	[1, 1, 1]	1.79402
[0, 4, 1]	[0, 1, 1]	1.80190	[2, 4, 2]	[1, 1, 1]	1.75008
[0, 4, 2]	[0, 1, 1]	1.75915	[3, 0, 2]	[2, 0, 2]	1.98235
[1, 1, 2]	[1, 0, 2]	1.98640	[3, 1, 0]	[3, 0, 0]	1.98640
[1, 2, 0]	[1, 1, 0]	1.98640	[3, 1, 1]	[3, 0, 1]	1.98640
[1, 2, 1]	[1, 1, 1]	1.98640	[3, 1, 2]	[2, 0, 2]	1.97733
[1, 2, 2]	[1, 0, 2]	1.95897	[3, 2, 0]	[2, 1, 0]	1.97733
[1, 3, 0]	[1, 1, 0]	1.95897	[3, 2, 1]	[3, 0, 1]	1.95897
[1, 3, 1]	[1, 1, 1]	1.95897	[3, 2, 2]	[2, 0, 2]	1.94997
[1, 3, 2]	[1, 1, 1]	1.91110	[3, 3, 0]	[2, 1, 0]	1.94997
[1, 4, 0]	[1, 1, 0]	1.80190	[3, 3, 1]	[2, 1, 0]	1.90118
[1, 4, 1]	[1, 1, 1]	1.80190	[3, 3, 2]	[1, 1, 1]	1.80885
[1, 4, 2]	[1, 1, 1]	1.75915	[3, 4, 0]	[2, 1, 0]	1.79402

#### 4.4 Solution Procedure

To solve the multi-mission selective maintenance scenario, an application was developed within Microsoft Excel. The user of the model inputs the following information:

- $m$  – number of subsystems
- $n_i$  – number of components in subsystem  $i$
- $r_i$  – reliability of a functioning type  $i$  component
- $s$  – number of limited maintenance resources
- $\alpha_{il}$  – amount of resource  $l$  required to repair a type  $i$  component
- $\beta_l$  – the amount of resource  $l$  available during each maintenance break

Visual Basic code within the Excel spreadsheet enumerates all possible combinations of failed components,  $\underline{a}$ . For each  $\underline{a} \in \Phi$ , the code generates a solution (i.e., the  $\underline{d}^*$ s). If the solution is infeasible, it is disregarded. However if a feasible solution is generated, the expected value of the number of successful missions with  $t$  missions remaining,  $W(t, \underline{a})$ , is tabulated. If the value of

$W(t,\underline{a})$  is equal to or greater than the largest previously computed value, that solution (and the corresponding value of the expected number of successful missions) is written to a file, and a new solution is generated. After generating all possible solutions for each mission in the scenario, the optimal solutions are output to an Excel worksheet. This enumeration code is able to handle any scenario size and guarantees the identification of optimal solutions. Also, solution feasibility checks and computations are done “on-the-fly” eliminating the excessive use of computer memory. Complete spreadsheet instructions for evaluating multi-mission scenarios are located in Appendix A.

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## Appendix A

File name: Multi\_Mission\_Solution\_Code

### 1. *Setting up the spreadsheet for use*

When you open the file, you may receive a message similar to the one shown in Figure A.1.

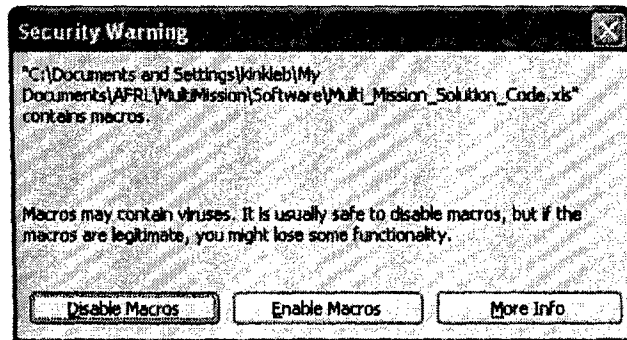


Figure A.1 Macro notification

The Visual Basic code used to evaluate the model is written within macros. Therefore, you should click on “Enable Macros.” This will open the Input worksheet shown in Figure A.2. Next, fill in the number of subsystems,  $m$ , and the number of limited maintenance resources,  $s$ , in the scenario you wish to evaluate. Once you have entered these values, activate a blank cell (by clicking on it), and click the “Reset Fields” button. Figure A.3 shows the updated Inputs worksheet for the scenario in which  $m = 2$  and  $s = 3$ .



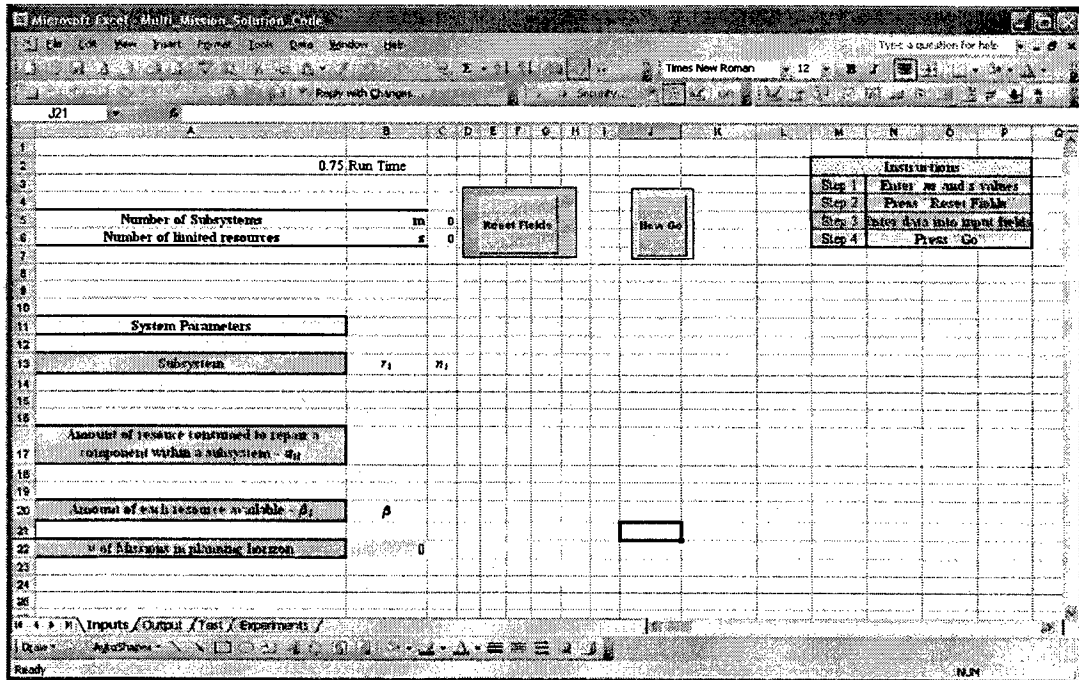


Figure A.2 Inputs worksheet

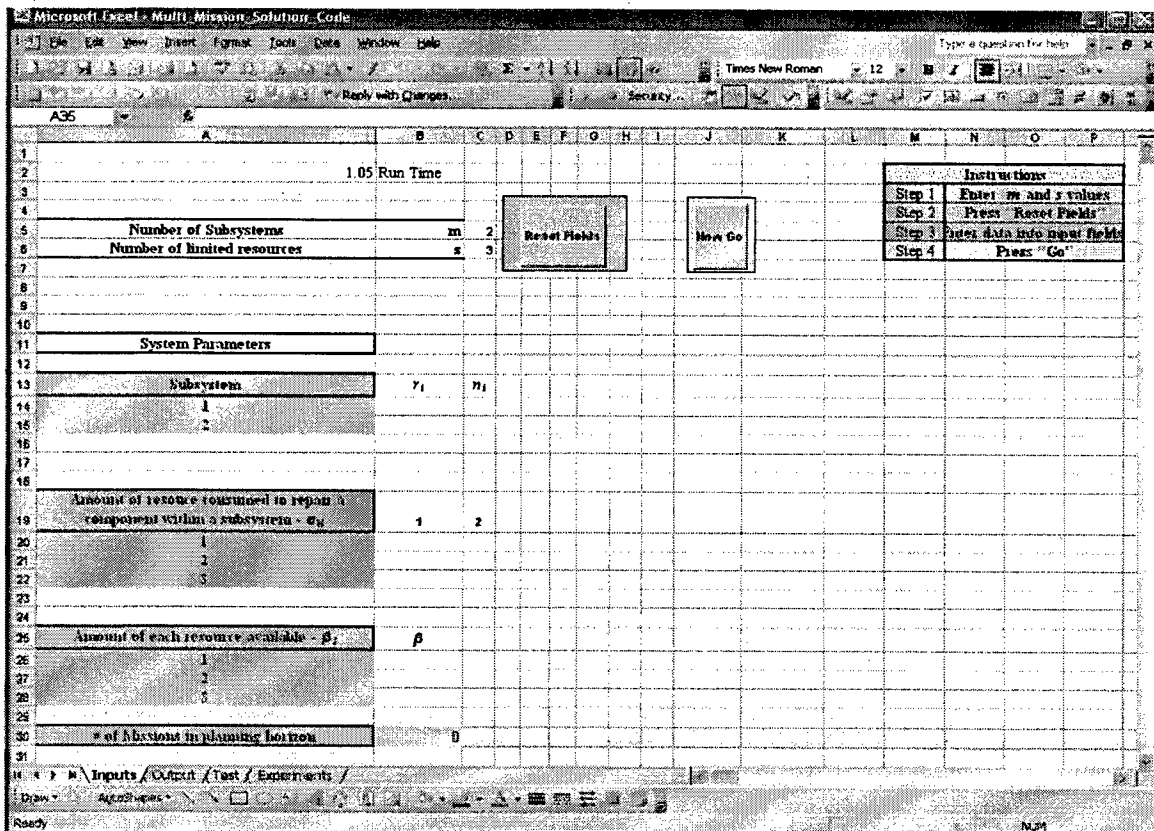


Figure A.3 Updated Inputs worksheet

## 2. Entering model parameter values

For each scenario you wish to evaluate, you must enter the model parameters. The first parameters of interest are the reliabilities of components in each subsystem,  $r_i$ , and the number of independent and identical copies of components in each subsystem,  $n_i$ . These values are input as shown in Figure A.4.

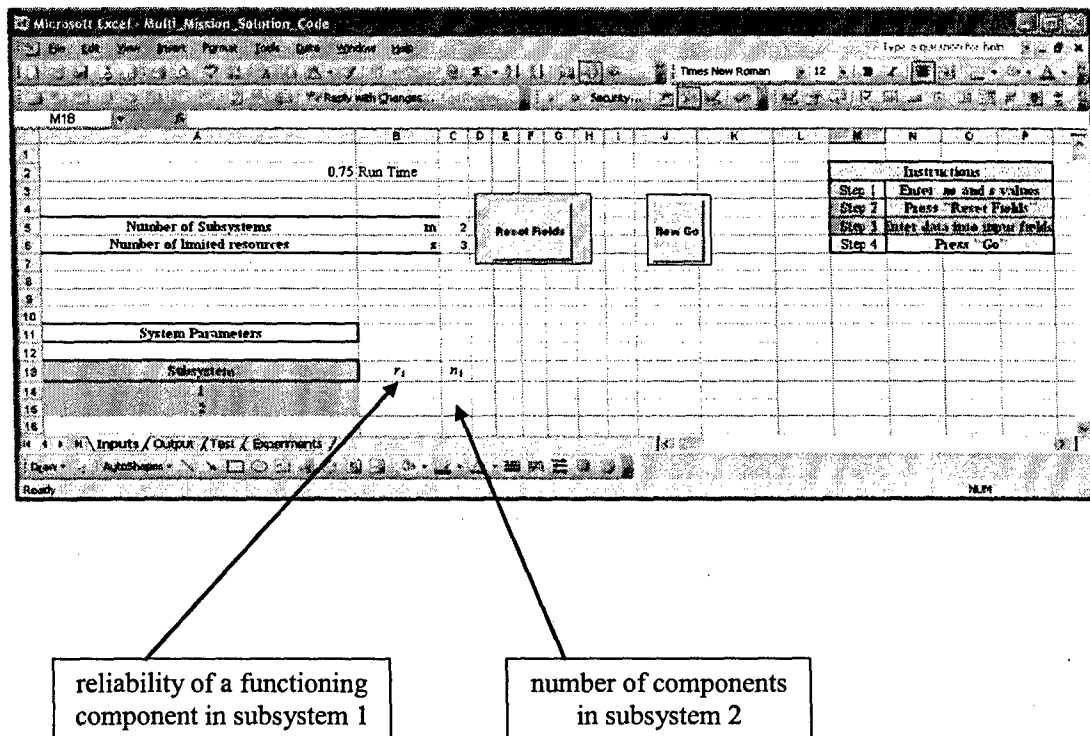


Figure A.4 Input parameters for component reliability and number of components

Now, you must input the amount of resource consumed by repairing a type  $i$  component,  $\alpha_{il}$ , the amount of each resource available,  $\beta_l$ , and the number of missions in the planning horizon,  $t$ . These values must be entered for each of the  $s$  limited maintenance resources (in this case,  $s = 3$ ) as shown in Figure A.5.

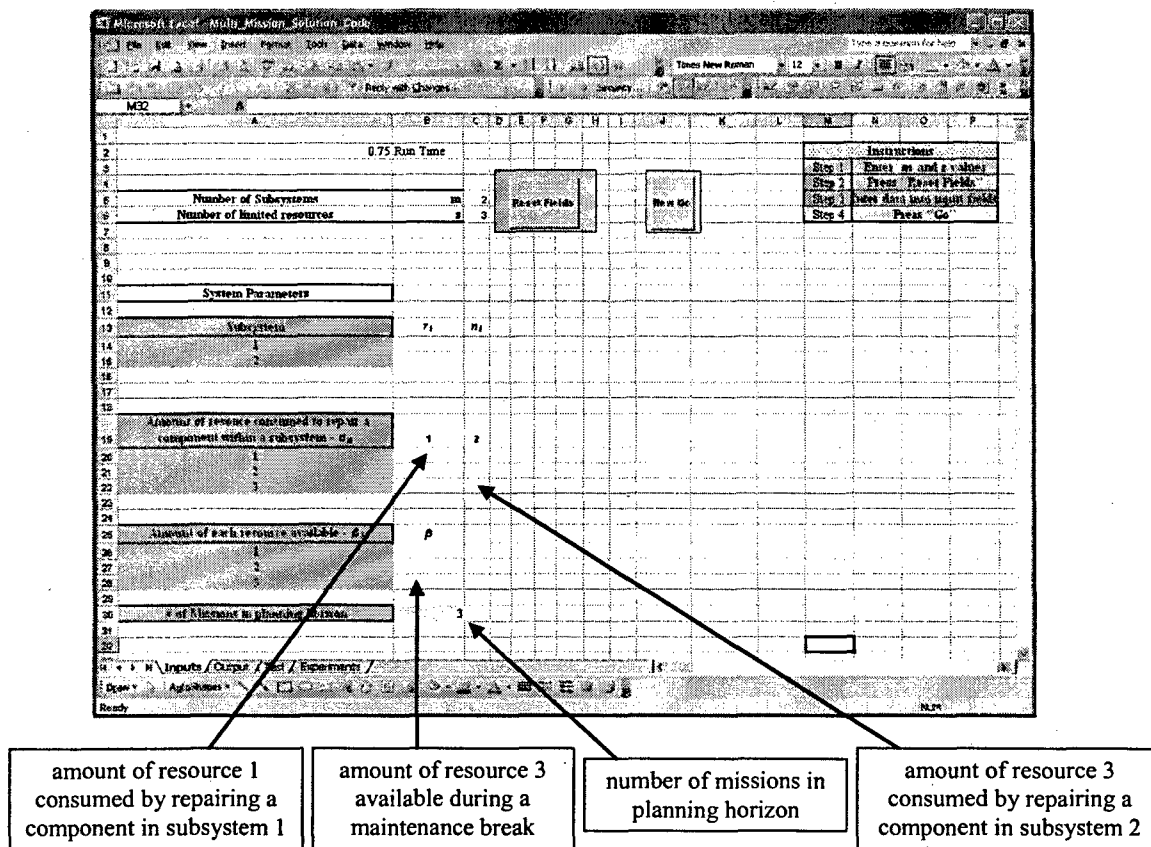


Figure A.5 Input parameters for limited maintenance resources

For a numerical example, suppose that  $m = 2$ ,  $s = 3$ ,  $t = 3$ ,  $\beta_1 = 12$ ,  $\beta_2 = 17$ ,  $\beta_3 = 17$ . (Note: It is imperative to have sufficient resources available to repair at least one failed component in each subsystem.) The remaining model parameters are shown in Table A.1. Figure A.6 provides a snapshot of the Inputs worksheet with the appropriate parameter inputs.

Table A.1 Model parameters for example system

Subsystem	$n_i$	$r_i$	$\alpha_{i1}$	$\alpha_{i2}$	$\alpha_{i3}$
1	5	0.95	1	8	2
2	8	0.85	3	9	5

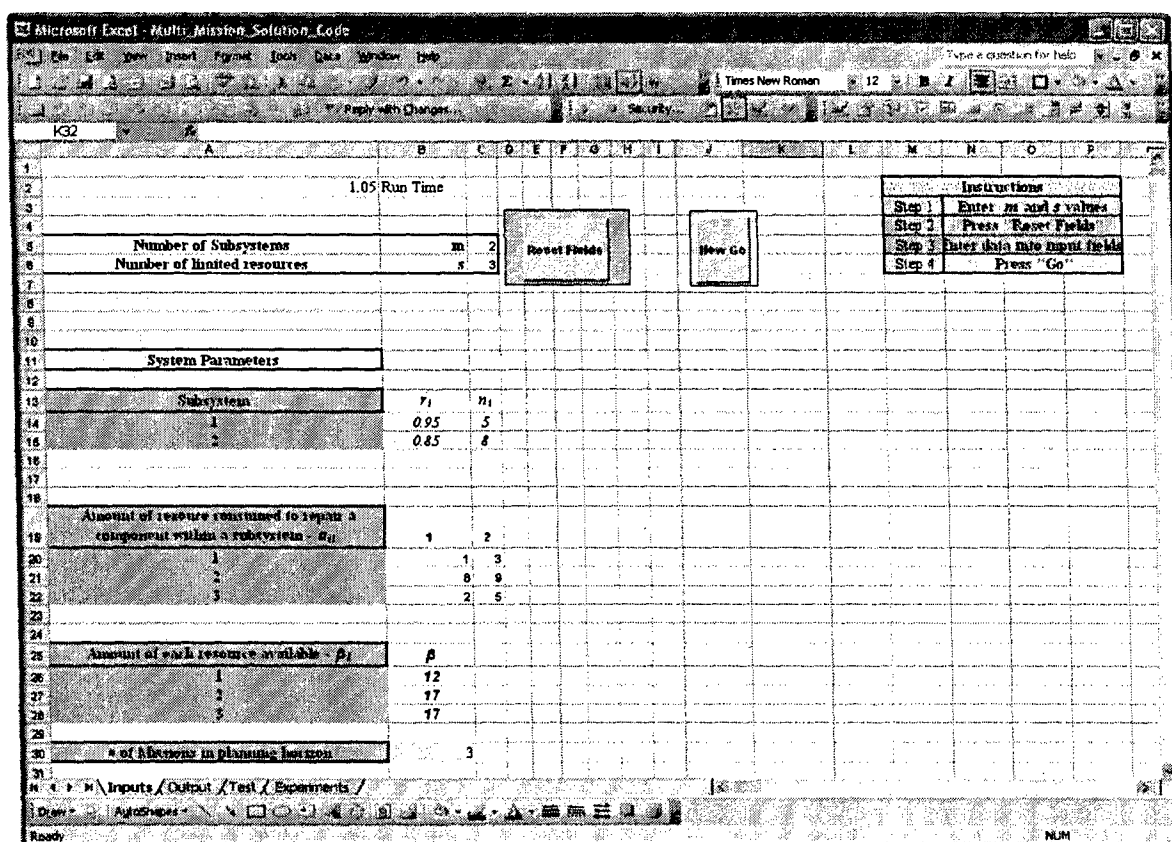


Figure A.6 Inputs worksheet with example parameter values

### 3. Running an experiment

Once you have input all of the model parameters, you are now ready to run an experiment. To run the model, simply activate a blank cell and click on the "New Go" button. The scenario instances that require selective maintenance are displayed in the "Output" worksheet. Initially, the results are displayed according to the number of missions remaining in the planning horizon, and a list is provided for instances where the single-mission solution differs from the multiple-mission solution. This is shown in Figure A.7. For this example, these instances are  $\underline{a} = [2, 1]$ ,  $\underline{a} = [3, 2]$ , and  $\underline{a} = [3, 3]$ .

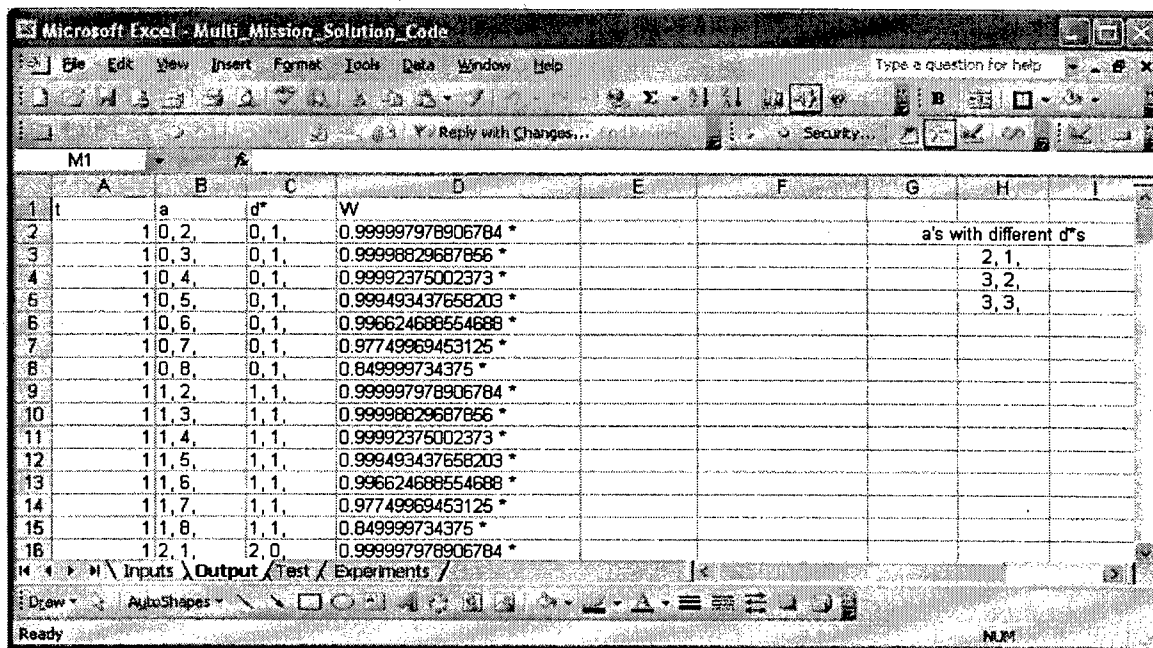


Figure A.7 Outputs worksheet for example scenario

The spreadsheet user may also sort the solutions by  $\underline{a}$  vector. To do this, simply highlight the area to be sorted, select “Data” then “Sort” from the toolbar. Then, sort the data as shown in Figure A.8.

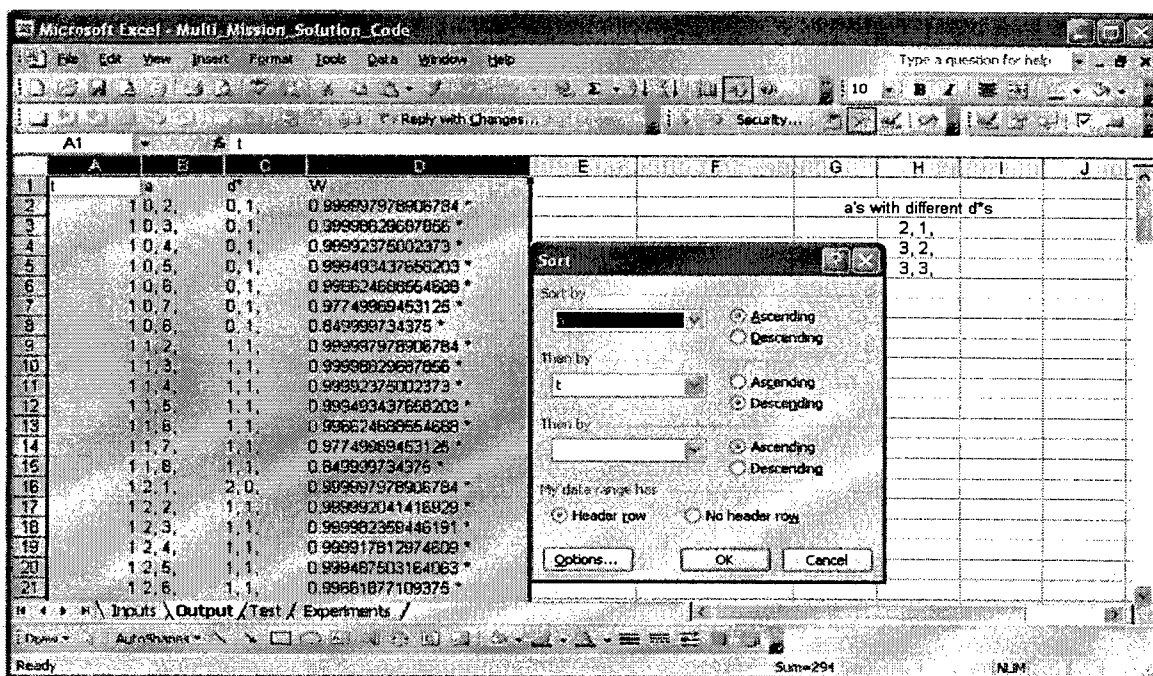


Figure A.8 Sorting the Outputs worksheet

Figure A.9 shows the sorted "Output" worksheet, and the scenario instances where the single-mission solution differs from the multiple-mission solution are highlighted.

	A	B	C	D	E	F	G	H	I
1	a	d*	W						
2	3,0,2,	0,1,	2.99992490905808 *				a's with different d's		
3	2,0,2,	0,1,	1.99997851877582 *				2,1		
4	1,0,2,	0,1,	0.999997978906784 *				3,2,		
5	3,0,3,	0,1,	2.99978124984196 *				3,3,		
6	2,0,3,	0,1,	1.99991927963635 *						
7	1,0,3,	0,1,	0.99996829687856 *						
41	3,1,8,	1,1,	2.79231125005232 *						
42	2,1,8,	1,1,	1.80837409201068 *						
43	1,1,8,	1,1,	0.849999734375 *						
44	3,2,1,	1,1,	2.99996434533736 *						
45	2,2,1,	1,1,	1.99998708972032 *						
46	1,2,1,	2,0,	0.999997978906784 *						
47	3,2,2,	1,1,	2.99991681107721 *						
48	2,2,2,	1,1,	1.99997139902686 *						
49	1,2,2,	1,1,	0.999992041416929 *						
50	3,2,3,	1,1,	2.99977244946566 *						
51	2,2,3,	1,1,	1.99991151397551 *						
52	1,2,3,	1,1,	0.999982359446191 *						
53	3,2,4,	1,1,	2.99931003217718 *						
54	2,2,4,	1,1,	1.99966792465138 *						
55	1,2,4,	1,1,	0.999917812974609 *						
56	3,2,5,	1,1,	2.9976696443235 *						
57	2,2,5,	1,1,	1.99859425005557 *						
58	1,2,5,	1,1,	0.999487503164063 *						
59	3,2,6,	1,1,	2.99101560346326 *						
60	2,2,6,	1,1,	1.99340911065419 *						
61	1,2,6,	1,1,	0.99661877109375 *						
62	3,2,7,	1,1,	2.95973859855781 *						
63	2,2,7,	1,1,	1.96593861450061 *						
64	1,2,7,	1,1,	0.977493890625 *						
65	3,2,8,	1,1,	2.7923012197533 *						
66	2,2,8,	1,1,	1.80836535714059 *						
67	1,2,8,	1,1,	0.8499946875 *						
68	3,3,0,	2,0,	2.99996434533736 *						
69	2,3,0,	2,0,	1.99998708972032 *						
70	1,3,0,	2,0,	0.999993493712539 *						
71	3,3,1,	2,0,	2.99991681107721 *						
72	2,3,1,	2,0,	1.99997139902686 *						
73	1,3,1,	2,0,	0.999992041416929 *						
74	3,3,2,	1,1,	2.99978161690294 *						
75	2,3,2,	2,0,	1.99991151397551 *						
76	1,3,2,	2,0,	0.999982359446191 *						
77	3,3,3,	1,1,	2.99963012942893 *						
78	2,3,3,	1,1,	1.99977853770524 *						
79	1,3,3,	2,0,	0.999917812974609 *						

Figure A.9 Sorted Outputs worksheet